Electron in the Vacuum of the Quantized Electromagnetic Field Using the Spherical Coordinates in Momentum Space

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Abstract— The quantization procedure of the Maxwell equation [1] was very successful, as it reduces to the quantization of an infinitive set of one-dimensional harmonic oscillators. The quantized field, multiplied by the electric charge, is interaction potential between the charged particle and photons. It perfectly describes the creation and annihilation of photons and is the base of quantum field theory, particularly quantum electrodynamics. The higher order perturbation terms give the divergent integrals, which after so called renormalization procedure precisely predict the small shifts of the atomic energy levels (the Lamb shift) and the anomaly magnetic moment of the electron.

In the few last decades, the quantum electrodynamics of radiative photonic environment is intensively studied [2].

In this work, we focus on the vacuum state of the quantized electromagnetic field, where no real photons are present. We use the spherical coordinate system for $\mathbf{k}$ vectors in momentum space (as opposed to coordinate space). Instead of using a cubic lattice, we adopt a spherical discretization scheme: the sphere is partitioned into concentric spherical shells (like an onion), which are further subdivided into parallel circular cross-sections. There are $4l$ states in each circle. All states on the spherical shell are degenerate — they all have the same energy. Then we can apply the orthogonal transform and if the state is the vacuum state and the vacuum is homogenous, we can reduce the number of the oscillators from $4l$ to l. As a consequence, the electric charge is effectively renormalized. The next step is to apply the unitary transformation to the parallel circles again and further reduce the number of oscillators. This mathematical approach may offer insight into the origin of divergent integrals in perturbation theory.

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REFERENCES

1. Dirac, P. A. M., *Proc. R. Soc.*, 114243, London, 1927.
2. Yuen, B. and A. Demetriadou, *Phys. Rev. Lett.*, Vol. 133, 203604, 2024.